Synthesis of Interstage Networks of Prescribed Gain Versus Frequency Slopes

DOUGLAS J. MELLOR, MEMBER, IEEE, AND JOHN G. LINVILL, FELLOW, IEEE

Abstract—Since the achievable gain of transistors typically falls off with increasing frequency, it is necessary to design interstage networks of microwave amplifiers with a complementary characteristic. A method is developed for the direct synthesis of interstage networks of prescribed gain versus frequency slopes. The bandwidth and ripple of these networks can also be precisely specified, and parasitic elements can be incorporated into synthesized networks. Design examples are presented, a complete computer program for the synthesis of interstage matching networks is described, and an octave band microwave amplifier illustrates an application of the interstage design techniques.

I. INTRODUCTION

TECHNIQUES have long been available for the direct synthesis of networks whose frequency response approximates a constant over a band of frequencies. In the synthesis procedure the insertion-loss function is first constrained to approximate a constant in equiripple or maximally flat fashion (the APPROXIMATION step). Once the insertion-loss function is defined, straightforward computational procedures are used to obtain a network whose frequency response is precisely that of the specifying insertion-loss function (the SYNTHESIS step).

In applications such as interstage design for microwave amplifiers, networks of sloped passband performance are needed to provide compensation for the gain roll-off of active devices with frequency and to yield an amplifier of overall flat transducer gain. Insertion-loss functions which approximate a specified gain versus frequency slope are therefore needed in the synthesis of interstage networks. Obtaining insertion-loss functions of prescribed gain versus frequency slopes is called here the SLOPED APPROXIMATION PROBLEM, and a general solution thereto has not been published. Ku and Petersen [1] have obtained sloped insertion-loss functions for low-pass topologies using curve-fitting techniques. The resultant passband ripple is unpredictable and must be reduced through an optimization process.

Here we develop a method of deriving insertion-loss functions with logarithmic gain versus frequency slopes from insertion-loss functions of flat passband performance. These insertion-loss functions exhibit exact logarithmic slopes for slopes which are integral multiples of 6 dB/octave (integral-sloped insertion-loss functions) and slightly distorted slopes otherwise (nonintegral-sloped insertion-loss functions). The slight distortion involved in the nonintegral-sloped insertion-loss functions is readily predictable from closed-form calculations. Using the insertion-loss functions of sloped passband performance, one obtains by standard synthesis techniques (the SYNTHESIS step) a network of prescribed bandwidth, gain slope, and ripple.

The synthesis approach reduces the interstage design problem to a straightforward computational procedure which is rapidly and accurately performed by a computer-aided design (CAD) program. An interactive computer routine implements the synthesis of matching networks to the user-specified passband, ripple, and desired gain versus frequency slope and adjusts the relative gain of the frequency response to insure inclusion of specified parasitic elements. The computer routine lists all available topologies, performs impedance transformations, and uses a transformed variable to improve numerical accuracy in the synthesis computation of element values.

A prototype 6.5–13-GHz amplifier illustrates an application of synthesized interstage networks for microwave amplifiers.

II. THE INTERSTAGE DESIGN PROBLEM

A. Characteristics of the Interstage Design Problem

Typical amplifier specifications call for a good input and output match and for an overall amplifier transducer gain which is constant (flat) over the passband. These specifications determine the impedance and frequency response characteristics of amplifier matching networks as illustrated in Fig. 1. The active devices are assumed unilateral at the outset, and the input and output impedances of these active devices are modeled in lumped-element form such that independent design of matching networks is possible. Thus each matching network operates between appropriate impedances [Fig. 1(b)] and must exhibit a flat or sloped frequency response as follows.

1) The specification for good input and output match implies a frequency response of the input and output matching networks that is flat over the passband [Fig. 1(d)].

2) The interstage matching networks (in general, there may be more than one) must provide a positive-sloped

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D. J. Mellor is with the Hewlett-Packard Company, Palo Alto, Calif. 94304.
J. G. Linvill is with the Department of Electrical Engineering, Stanford University, Stanford, Calif. 94305.

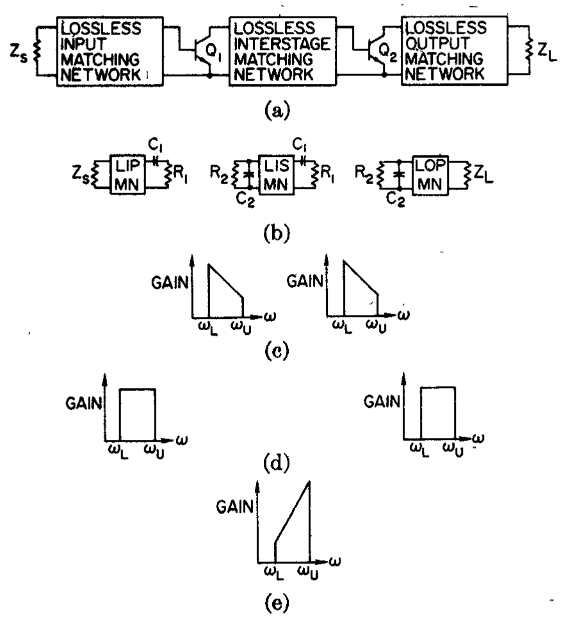

Fig. 1. Characteristics of a typical amplifier design problem after the unilateral and lumped-element impedance idealizations are made. (a) Amplifier schematic. (b) Impedances. (c) Frequency response of transistors. (d) Frequency response of input and output matching networks. (e) Frequency response of interstage matching network.

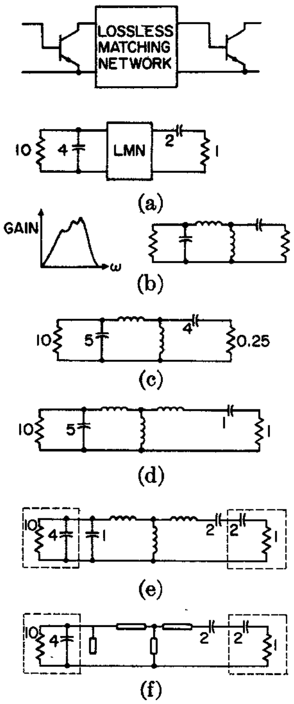

Fig. 2. Outline of matching synthesis procedure. (a) Model device impedances. (b) Constrain frequency response and select topology consistent with parasitic elements. (c) Synthesize network. (d) Transform impedance. (e) Separate out device impedances. (f) Transform design to transmission-line equivalent.

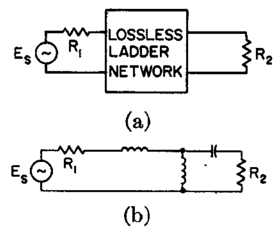

Fig. 3. (a) General definition of trapless filters. (b) Example of a trapless filter.

gain with frequency [Fig. 1(e)] to compensate the transistors' rolloff [Fig. 1(c)] and give an overall flat transducer gain.

B. Solution to the Interstage Design Problem: MATCHING SYNTHESIS

The procedure described here for the synthesis of matching networks of arbitrarily specified passband slope with provision for inclusion of parasitic elements is called MATCHING SYNTHESIS [2] and is outlined in Fig. 2. A key part of this design procedure is the synthesis of interstage networks with specified gain versus frequency slopes.

III. THE SYNTHESIS OF MATCHING NETWORKS OF SLOPED PASSBAND RESPONSE

A. Definition of Synthesis Terms

1) Trapless Filters: Two resistors coupled by a lossless ladder network having all transmission zeros at zero and infinite frequency are here called trapless filters. The insertion loss for such filters can be expressed as follows [see Fig. 3(a)]:

$$\text{insertion loss} = \text{IL} = \frac{\text{power available from } R_1, E_s}{\text{power delivered to } R_2}$$

$$= \frac{a_0 + a_2\omega^2 + \cdots a_{2N}\omega^{2N}}{\omega^{2J}} \qquad (1)$$

N order of network = number of natural frequencies of the network;

J number of transmission zeros at zero frequency;

$N - J$ number of transmission zeros at infinite frequency.

The example of Fig. 3(b) serves to clarify these definitions. The insertion loss for the network of Fig. 3(b) is

$$\text{IL} = \frac{a_0 + a_2\omega^2 + a_4\omega^4 + a_6\omega^6}{\omega^4} \quad N = 3 \quad J = 2.$$

We restrict our attention throughout this paper to the synthesis of trapless filters as matching networks.

2) The Synthesis Process: Passive network synthesis is a well-known procedure for obtaining networks of prescribed frequency response [3]–[5]. The synthesis procedure consists of two steps [Fig. 4(a)].

a) The APPROXIMATION Step: An insertion-loss function is first obtained which approximates a flat passband response in equiripple or maximally flat fashion.

b) The SYNTHESIS Step: Straightforward computational procedures yield a network as prescribed by the insertion-loss function.

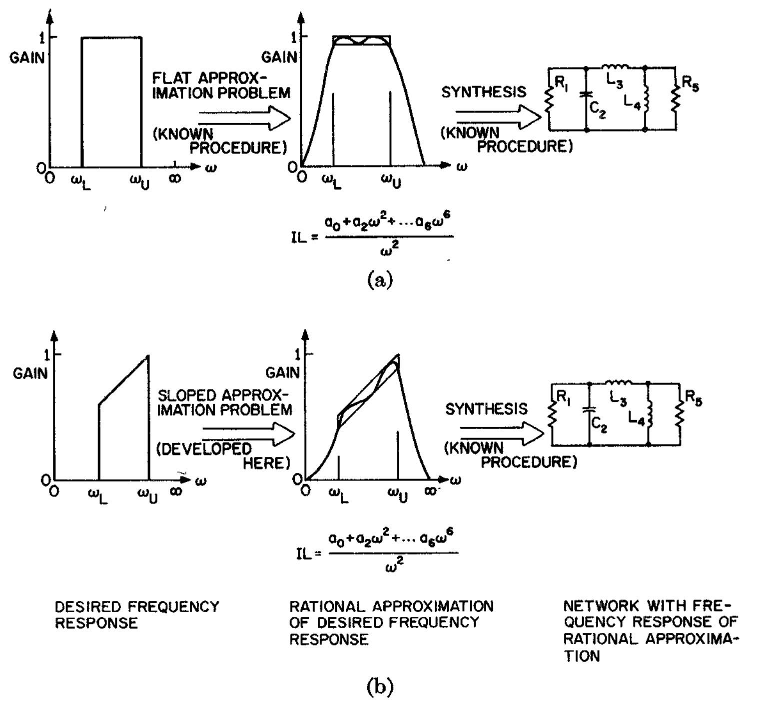

Fig. 4. (a) The familiar procedure of synthesizing a network of flat passband performance. (b) The sloped approximation problem is solved to yield networks of sloped passband performance.

B. The SLOPED APPROXIMATION PROBLEM

The synthesis of interstage networks requires that insertion-loss functions of specified gain versus frequency slope, bandwidth, and ripple be found (the SLOPED APPROXIMATION PROBLEM) as illustrated in Fig. 4(b). Once the sloped insertion-loss function is obtained, the familiar SYNTHESIS step yields a network having the specified frequency response.

C. The Derivation of Integral-Sloped Insertion-Loss Functions: Multiples of 6 dB/Octave

Insertion-loss functions approximating $6S$-dB/octave gain slope, where S is an integer, are easily obtained from flat insertion-loss functions. The flat insertion-loss function is first normalized to an upper cutoff frequency of 1 rad/s and then divided by ω^{2S}. This operation, shown in Fig. 5, results in a sloped insertion loss of exactly the same ripple and passband as the flat insertion loss, the bounds being exact logarithmic curves of $K_0\omega^{-2S}$. The order of the insertion-loss function remains unchanged, while the number of transmission zeros at dc (J) is increased by S.

All that is required for the generation of an integral-sloped insertion loss-function with $N,J = N_S,J_S$ then, is a flat insertion-loss function with $N,J = N_S,J_S - S$. The flat insertion-loss function becomes the desired sloped insertion-loss function upon division by ω^{2S}.

D. The Derivation of Nonintegral-Sloped Insertion-Loss Functions: Nonmultiples of 6 dB/Octave

Nonintegral-sloped insertion-loss functions are obtained by a linear combination of two insertion-loss functions

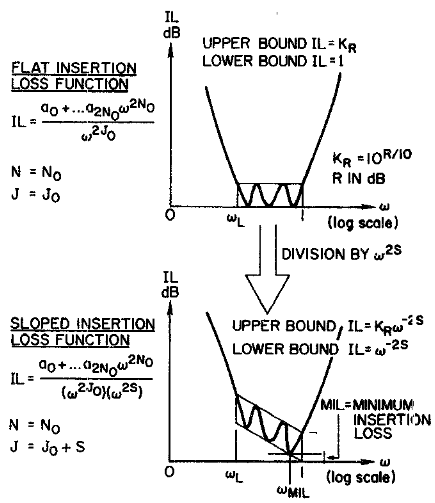

Fig. 5. Division by ω^{2S} (where S is an integer) results in an exact logarithmically sloped insertion-loss function.

having an integral slope just less than, and just greater than, the desired slope. The resultant combination approximates the desired nonintegral slope with a slight deviation from true logarithmic slope. The amount of deviation is readily predictable.

We will combine integral-sloped insertion-loss functions of S_1 and S_2 slope which have the same ripple and the same N and J. S_1 and S_2 are defined as follows:

$$S = \text{desired nonintegral slope}$$

$$S_1 = \text{INT}(S) = \text{integer just smaller than } S$$

$$S_2 = S_1 + 1 = \text{integer just greater than } S. \quad (2)$$

Since we are able only to approximately obtain the desired nonintegral slope, we constrain the bounds of the insertion loss to be exactly as desired at the passband edges and check the resultant deviation within the passband. We therefore seek the combining constants A_1 and A_2 of Fig. 6 which will set the bounds of the approximation properly at the band edges. The four conditions on A_1 and A_2 are as follows. The condition on the lower bound at $\omega = \omega_L$ is

$$A_1\omega_L^{-2S_1} + A_2\omega_L^{-2S_2} = \omega_L^{-2S}. \tag{3a}$$

The condition on the lower bound at $\omega = \omega_U = 1$ is

$$A_1 + A_2 = 1. \tag{3b}$$

The condition on the upper bound at $\omega = \omega_L$ is

$$A_1K_R\omega_L^{-2S_1} + A_2K_R\omega_L^{-2S_2} = K_R\omega_L^{-2S}$$

$$K_R = 10^{R/10} \qquad R \text{ in decibels.} \tag{4a}$$

The condition on the upper bound at $\omega = \omega_U = 1$ is

$$K_RA_1 + K_RA_2 = K_R. \tag{4b}$$

By inspection, (4a) and (4b) are fulfilled if (3a) and (3b) are satisfied. The solution to (3) is

$$A_1 = \frac{\omega_L^{2(S_2-S)} - 1}{\omega_L^2 - 1} \qquad A_2 = 1 - A_1. \tag{5}$$

The deviation of the resultant insertion-loss function from true logarithmic slope is described by Fig. 6 and can be defined as follows:

$$\text{deviation} = \frac{\text{bounds of actual response}}{\text{bounds of desired response}}$$

$$= \frac{A_1\omega^{-2S_1} + A_2\omega^{-2S_2}}{\omega^{-2S}}. \tag{6}$$

The frequency of maximum deviation ω_{MD} and the maximum deviation (MD), can be found by taking the derivative of (6) with respect to ω:

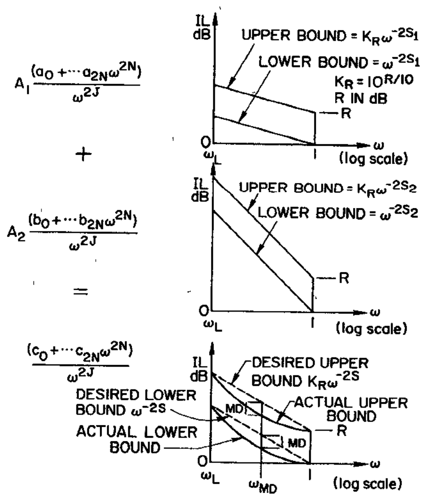

Fig. 6. The linear combination of integral-sloped insertion-loss functions results in a nonintegral-sloped insertion-loss function.

$$\frac{d\text{ deviation}}{d\omega} = 0 \Rightarrow \omega_{MD}^2 = \frac{A_2(S_2 - S)}{A_1(S - S_1)}. \tag{7}$$

E. Flat Insertion-Loss Functions Used for the Generation of Sloped Insertion-Loss Functions

In order to afford maximum flexibility in matching network designs, it is desirable to have the greatest possible flexibility in the choice of N and J (the order and the number of transmission zeros at dc, respectively). Thus flat insertion-loss functions of arbitrary N and J are needed from which to derive sloped insertion-loss functions. We make use of Szentirmai's insertion-loss functions [6] of flat passband performance from which to derive sloped insertion-loss functions. Since Szentirmai's insertion-loss functions allow

$$N_0 = \text{arbitrary}$$

$$0 \leq J_0 \leq N_0.$$

Sloped insertion-loss functions are possible for

$$\left.\begin{array}{l} N_S = \text{arbitrary} \\ S \leq J_S \leq N_S \end{array}\right\} \text{for integer slopes}$$

and

$$\left.\begin{array}{l} N_S = \text{arbitrary} \\ \text{INT}(S) + 1 \leq J_S \leq N_S \end{array}\right\} \text{for noninteger slopes.}$$

The subscripts 0 and S refer to the flat and sloped insertion-loss functions, respectively. The fact that Szentirmai's techniques for obtaining flat insertion-loss functions are cast in a $\emptyset$-plane formulation is immaterial since all the operations needed to obtain sloped insertion-loss functions can be performed equally well in the $\emptyset$ plane. The $\emptyset$ variable is a transformation from the familiar s variable via the following equation:

$$\emptyset^2 = \frac{(s/\omega_0)^2 + B}{B(s/\omega_0)^2 + 1}, \qquad B = \omega_U/\omega_L, \quad \omega_0^2 = \omega_U\omega_L$$

$$\omega_L = \text{lower cutoff frequency}$$

$$\omega_U = \text{upper cutoff frequency.} \tag{8}$$

The use of this transformed variable for improved numerical accuracy in the synthesis process is well known [6]–[9].

F. The Adjustment for Desired Minimum Insertion Loss of Sloped Insertion-Loss Functions

Since the sloped insertion-loss functions were derived by putting a slant on appropriate flat insertion-loss functions, their minimum insertion loss (MIL) will, in general, be nonzero (see Fig. 5). The actual MIL of both integral- and nonintegral-sloped approximations is therefore unpredictable and should be adjusted to a value specified by the designer. Adjustment of the MIL to a value of 0 dB or any other desired value requires finding the frequency at which the MIL occurs:

$$\frac{d\text{IL}}{d\omega} = 0 \Rightarrow \omega_{\text{MIL}} \tag{9}$$

$$\text{IL}(\omega_{\text{MIL}}) = \text{MIL}. \tag{10}$$

After the frequency ω_{MIL} and the corresponding MIL have been determined, the insertion-loss function is adjusted to a desired MIL, MIL_{des}, by constant multiplication (MIL_{des} and MIL in decibels):

$$\text{IL}_{\text{des}} = \text{IL} \times K_{\text{MILSET}} \tag{11}$$

$$K_{\text{MILSET}} = 10^{(\text{MIL}_{\text{des}} - \text{MIL})/10}. \tag{12}$$

G. Inclusion of Parasitic Elements

The presence of parasitic elements limits the available gain bandwidth of a matching network [10], and this available gain bandwidth product must not be exceeded in the synthesis specification if inherent parasitic elements are to be absorbed into synthesized networks. Since the bandwidth and ripple of a design are generally fixed, the gain of the specifying insertion-loss function is adjusted to ensure absorption of parasitic elements[1]:

$$\text{IL}_P = K^2\text{IL}. \tag{13}$$

This adjustment procedure is illustrated in our examples.

IV. THE SYNTHESIS OF MATCHING NETWORKS OF SLOPED PASSBAND PERFORMANCE: NUMERICAL EXAMPLES

A. Introduction

Two numerical examples are given here to illustrate how sloped insertion-loss functions can be obtained from suitable flat insertion-loss functions, thereby enabling the synthesis of networks of specified gain versus frequency slope. The first example is an integral-sloped insertion-loss function calculated in the s plane using familiar approximation techniques. The second example is a derivation of a nonintegral-sloped insertion-loss function from $\emptyset$-plane insertion-loss functions of flat passband.

B. Example: Derivation of Integral-Sloped Approximation: s Plane

An interstage network is to operate between a 50-Ω source and a series 10-Ω and 0.82-pF load. A 6-dB/octave gain slope and 1 dB of ripple are required in a 1–2-GHz passband.

Outline of Solution: First, a flat insertion loss (IL_F) will be derived using standard approximation techniques, then this insertion loss will be divided by ω^2 to obtain a sloped insertion loss (IL_S), and finally the sloped insertion loss is multiplied by a constant to obtain an insertion loss that will absorb the prescribed parasitic (IL_{SP}). From IL_{SP} the network will be synthesized.

Solution:

$$\text{IL}_F = 1 + K_1C_2^2(K_2(\omega/\omega_0 - \omega_0/\omega))$$

$C_2(X)$ = second-order Chebyshev polynomial = $2X^2 - 1$.

The frequency normalization is taken such that the upper cutoff frequency is scaled[2] to 1 rad/s:

$$\omega_U = 1 \text{ rad/s} \quad \omega_L = 0.5 \text{ rad/s}$$

$$\omega_0 = (\omega_U\omega_L)^{1/2} = 0.707 \text{ rad/s}.$$

K_2 sets the relative bandwidth:

$$K_2(\omega_U/\omega_0 - \omega_0/\omega_U) = 1$$

$$\therefore K_2 = 1.414.$$

K_1 adjusts the ripple: $K_1 = 10^{0.1} - 1 = 0.26$

$$\text{IL}_F = \frac{16.6\omega^8 - 37.3\omega^6 + 30.3\omega^4 - 9.3\omega^2 + 1.04}{\omega^4}, \quad (N,J = 4,2).$$

The flat insertion loss obtains 6-dB/octave slope upon division by ω^2:

$$\text{IL}_S = \text{IL}_F/\omega^2 = \frac{16.6\omega^8 - 37.3\omega^6 + 30.3\omega^4 - 9.3\omega^2 + 1.04}{\omega^6}, \quad (N,J = 4,3)$$

$$\text{IL}_{SP} = K^2\text{IL}_S.$$

The value of $K^2 = 1.26$ is sufficient to ensure absorption of the specified parasitic

$$\text{IL}_{SP} = \frac{20.9\omega^8 - 46.9\omega^6 + 38.1\omega^4 - 11.7\omega^2 + 1.3}{\omega^6}.$$

The network synthesized from this insertion loss after impedance transformation [11] and scaling to 2-GHz upper cutoff is shown in Fig. 7 along with the plotted response of the network (IL_{SP}). IL_F and IL_S are plotted also for comparison.

C. Example: Derivation of Nonintegral-Sloped Insertion-Loss Functions: $\emptyset$ Plane

An interstage is to be designed to operate between a 200-Ω source and a 10-Ω load with 0.5 dB of ripple and 4-dB/octave slope in a 1–2-GHz passband. Minimum insertion loss is required.

Outline of Solution: A flat insertion-loss function (IL_{S0}) will be obtained and divided by ω^2 to obtain a 6-dB/octave insertion-loss function (IL_{S6}). This 6-dB/octave insertion-loss function will be linearly combined with yet another flat insertion-loss function (IL_{F0}) to obtain an insertion-loss function with 4-dB/octave slope (IL_{S4}). Finally, the

[1] A more complete discussion of adjusting the frequency response specification in order to ensure inclusion of parasitic elements is contained in [2].

[2] All numerical results are shown here with less accuracy than would be necessary for accurate computations.

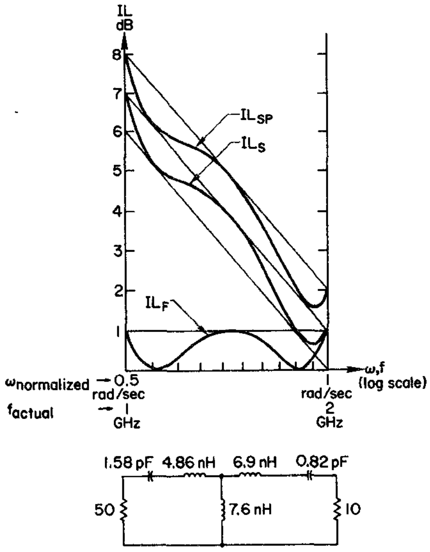

Fig. 7. A matching network of 6-dB/octave slope synthesized from IL_{SP}.

4-dB/octave insertion-loss function is adjusted for minimum insertion loss by constant multiplication. The adjusted insertion loss is IL_{S4A}. From IL_{S4A} the network is synthesized. In equation form:

$$\underset{\begin{Bmatrix}\text{flat}\\ N,J=4,1\end{Bmatrix}}{IL_{S0}} \div \omega^2 = \underset{\begin{Bmatrix}6\text{ dB/octave}\\ N,J=4,2\end{Bmatrix}}{IL_{S6}}$$

$$\underset{\begin{Bmatrix}\text{flat}\\ N,J=4,2\end{Bmatrix}}{A_1 IL_{F0}} + \underset{\begin{Bmatrix}6\text{ dB/octave}\\ N,J=4,2\end{Bmatrix}}{A_2 IL_{S6}} = \underset{\begin{Bmatrix}4\text{ dB/octave}\\ N,J=4,2\end{Bmatrix}}{IL_{S4}}$$

$$\underset{\begin{Bmatrix}4\text{ dB/octave}\\ N,J=4,2\end{Bmatrix}}{IL_{S4}} \times K_{\text{MILSET}} = \underset{\begin{Bmatrix}4\text{ dB/octave}\\ N,J=4,2\end{Bmatrix}}{IL_{S4A}}.$$

Solution: For all insertion-loss functions in the $\emptyset$-plane formulation, the following frequency normalization is used:

$$\omega_U = 1 \quad \omega_L = 0.5 \quad \omega_0 = \omega_U\omega_L = 0.707$$

$$B = \omega_U/\omega_L = 2.$$

Szentirmai's $\emptyset$-plane approximation techniques [6] are used to generate a flat insertion-loss function to the following specifications: 0.5-dB ripple, $N = 4$, $J = 1$

$$IL_{S0} = \frac{1.12 - 4.3\emptyset^2 + 25.4\emptyset^4 - 9.6\emptyset^6 + 4.5\emptyset^8}{1 - 6.5\emptyset^2 + 15\emptyset^4 - 14\emptyset^6 + 4\emptyset^8}, \qquad (N,J = 4,1).$$

A 6-dB/octave slope is introduced upon division by ω^2, where ω^2 in the $\emptyset$ plane can be obtained by rearranging (8):

$$\omega^2 = -s^2 = \frac{\omega_0^2(B - \emptyset^2)}{(B\emptyset^2 - 1)} = \frac{(2 - \emptyset^2)}{2(2\emptyset^2 - 1)}$$

$$IL_{S6} = IL_{S0}/\omega^2$$

$$IL_{S6} = \frac{\dfrac{1.12 - 4.3\emptyset^2 + 25.4\emptyset^4 - 9.6\emptyset^6 + 4.5\emptyset^8}{(1 - 6.5\emptyset^2 + 15\emptyset^4 - 14\emptyset^6 + 4\emptyset^8)(2 - \emptyset^2)}}{2(2\emptyset^2 - 1)}$$

$$IL_{S6} = \frac{1.12 - 4.3\emptyset^2 + 25.4\emptyset^4 - 9.6\emptyset^6 + 4.5\emptyset^8}{1 - 5\emptyset^2 + 8.25\emptyset^4 - 5\emptyset^6 + \emptyset^8}, \qquad (N,J = 4,2).$$

Now a flat insertion-loss function with $N = 4$, $J = 2$, and 0.5-dB ripple is obtained:

$$IL_{F0} = \frac{1.12 - 3.4\emptyset^2 + 13.6\emptyset^4 - 3.4\emptyset^6 + 1.12\emptyset^8}{1 - 5\emptyset^2 + 8.25\emptyset^4 - 5\emptyset^6 + \emptyset^8}, \qquad (N,J = 4,2).$$

IL_{F0} (zero slope) and IL_{S6} (6-dB/octave slope) are now combined to obtain 4-dB/octave slope:

$$IL_{S4} = A_1 IL_{F0} + A_2 IL_{S6}.$$

Equation (5) gives

$$\begin{matrix} A_1 = 0.49 \\ \\ A_2 = 0.51 \end{matrix} \qquad \text{for} \begin{cases} \omega_L = 0.5 \\ S_2 = 1 \qquad S_1 = 0 \\ S = 2/3 \end{cases}$$

$$IL_{S4} = \frac{1.12 - 3.8\emptyset^2 + 19.6\emptyset^4 - 6.6\emptyset^6 - 2.8\emptyset^8}{1 - 5\emptyset^2 + 8.25\emptyset^4 - 5\emptyset^6 + \emptyset^8}, \qquad (N,J = 4,2).$$

Fig. 8 displays the frequency response of IL_{S4}. The frequency of MD and the MD (maximum departure from 4-dB/octave slope) are obtained via (6) and (7):

$$\omega_{MD} = 0.725 \quad MD = 0.2 \text{ dB}.$$

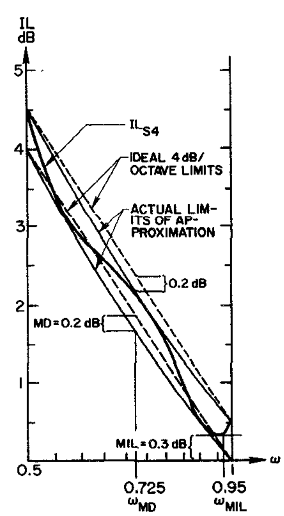

Fig. 8. The frequency response of the 4-dB/octave insertion-loss function.

The frequency of MIL and the MIL are computed for IL_{S4}

$$\frac{dIL_{S4}}{d\emptyset} = 0 \Rightarrow \emptyset_{MIL}{}^2 = -0.0683$$

$$\omega_{MIL} = 0.95 \quad MIL = 0.3 \text{ dB}.$$

IL_{S4} is adjusted for maximum gain by constant multiplication

$$IL_{S4A} = K_{MILSET} \times IL_{S4}$$

$$K_{MILSET} = 10^{-0.3/10} = 0.933$$

$$IL_{S4A} = \frac{1.05 - 3.6\emptyset^2 + 18.3\emptyset^4 - 6.1\emptyset^6 + 2.6\emptyset^8}{1 - 5\emptyset^2 + 8.25\emptyset^4 - 5\emptyset^6 + \emptyset^8}, \quad (N,J = 4,2).$$

IL_{S4A} is plotted in Fig. 9. The matching network obtained from IL_{S4A} using $\emptyset$-plane synthesis after frequency scaling and impedance transformation [11] is also shown in Fig. 9.

V. CAD IMPLEMENTATION OF MATCHING SYNTHESIS

The rapid and accurate design of matching networks by synthesis methods is best accomplished by an interactive CAD program. The computer-aided implementation of MATCHING SYNTHESIS [2] represents a unique total package for synthesis of matching networks including the following:

1) synthesis of networks of prescribed ripple, bandwidth, and gain versus frequency slope;
2) adjustment of the frequency response to assure inclusion of parasitic elements into synthesized networks;
3) generation of all allowable topologies for a given synthesis specification;
4) time-shared synthesis capability up to twelfth order utilizing $\emptyset$-plane synthesis;
5) automated implementation of impedance transformations.

VI. AMPLIFIER PROTOTYPE

The matching networks for a prototype amplifier covering 6.5–13 GHz were designed using the interstage design methods of this paper. A block diagram and a photograph of the amplifier using the HP GaAs FET [12], [13] is shown in Fig. 10. The characteristics of the individual blocks of the amplifier are as follows.

HP GaAs FET Characteristics

Maximum available gain (G_{amax}):	14.75 dB at 6.5 GHz	Approximate 6-dB/octave rolloff
	8.62 dB at 13 GHz	

Input impedance: modeled as a series R–C network
Output impedance: modeled as a shunt R–C network

Input Matching-Network Characteristics

Gain versus frequency slope:	6 dB/octave	over 6.5–13 GHz
MIL:	0.0 dB	
Response:	maximally flat	
Curvature:	0.3 dB	

Output Matching-Network Characteristics

Gain versus frequency slope:	0.0 dB/octave (flat)	over 6.5–13 GHz
MIL:	0.0 dB	
Response:	equiripple	
Ripple:	0.7 dB	

The measured small-signal performance of this thin-film amplifier is shown in Fig. 11. The ideal gain curve is that predicted for the amplifier based on a unilateral assumption for the FET ($S_{12} = 0$) and simple lumped-element models of the FET input and output impedances as well as for the matching networks. The close agreement between the ideal and measured response verifies that octave band microwave amplifiers are readily designed using synthesized matching networks.

Because of the mismatch at the input, the amplifier is

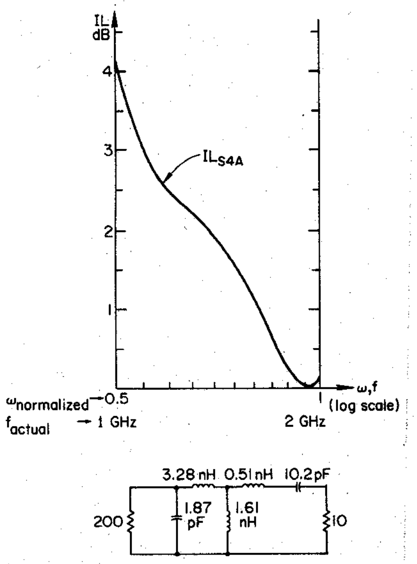

Fig. 9. The 4-dB/octave sloped insertion-loss function is adjusted for maximum gain and a network is synthesized from IL_{S4A}.

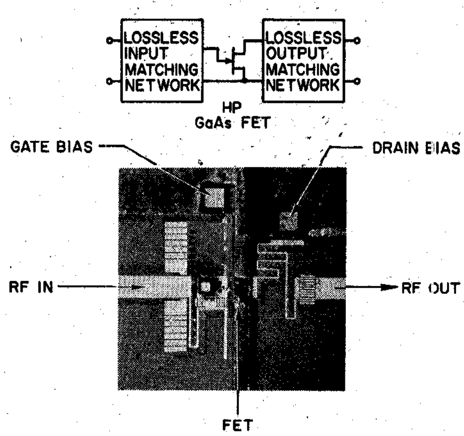

Fig. 10. Block diagram and photograph of prototype 6.5–13-GHz amplifier.

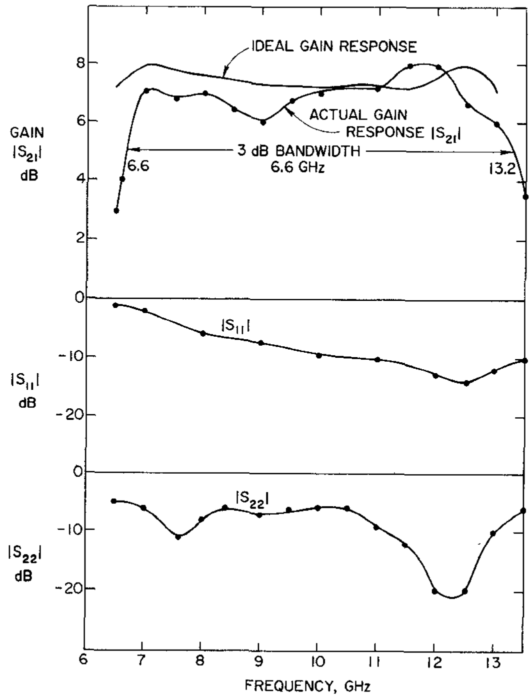

Fig. 11. Measured response of a prototype 6.5–13-GHz amplifier.

not suited for direct cascading with other amplifiers or other system building blocks, but is an excellent unit amplifier for hybrid-coupled [14] or circulator-coupled systems.

VII. CONCLUSION

The powerful tool of passive network synthesis has been generalized to include the synthesis of networks with arbitrary gain versus frequency slopes and to allow the inclusion of parasitic elements into synthesized networks. With this generalization, interstage networks for microwave amplifiers can be readily designed using straightforward synthesis methods. Since the synthesis process is a step-by-step computational procedure, it is readily amenable to CAD programming. From a user-specified bandwidth, gain versus frequency slope, and ripple, a CAD program provides the configurations and element values of the matching networks and implements impedance transformations necessary for proper termination. Thus interstage design is accomplished rapidly and accurately through direct synthesis methods.

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